УДК 512.7

## E. I. Smirnov

## The Category of Hausdorff Spectra over a Semiabelian Category

In this article the category  $\mathcal{H}$  of Hausdorff spectra is introduced into the discussion by means of an appropriate factorization of the category of Hausdorff spectra Spect  $\mathcal{G}$  over the category  $\mathcal{G}$ . If  $\mathcal{G}$  is a semiabelian complete subcategory of the category TG, then  $\mathcal{H}$  is a semiabelian category (in the sense of V. P. Palamodov [1]).

Key words: Hausdorff spectra, semiabelian category, commutative diagram.

Let  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\}$  and  $\mathcal{Y} = \{Y_p, \mathfrak{F}^1, h_{p'p}\}$  be Hausdorff spectra over some category  $\mathcal{G}$ . We will call any set of morphisms  $\omega_{ps} : X_s \to Y_p$  of the category  $\mathcal{G}$  which satisfies the following conditions a *mapping of spectra*  $\omega_{\mathcal{YX}} : \mathcal{X} \to \mathcal{Y}$ :

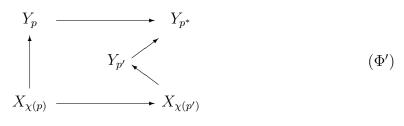
(1) there exist mappings  $\varphi : \mathfrak{F} \to \mathfrak{F}^1$ ,  $\phi : F^1 \to F \; (\forall F \in \mathfrak{F}), \; \chi^{T^1} : T^1 \to T \; (\forall T^1 \in F^1), \; \chi^{T^1}|_{T^1_0} = \chi^{T^1_0}, \; T^1_0 \subset T^1 \text{ such that } (\mapsto \text{ denotes mapping of elements})$ 

s	$\in$	T	$\in$	F	$\in$	F
$\chi$		$\chi     \Phi$		$\Phi {\begin{bmatrix} \varphi \\ \varphi $		$\varphi$
p	$\in$	$T^1$	$\in$	$F^1$	$\in$	$\mathfrak{F}^1$

(2) for each pair  $(p, \chi(p))$  a morphism  $\omega_{p\chi(p)} : X_{\chi(p)} \to Y_p$  of the category  $\mathcal{G}$  is defined in such a way that if  $h_{p^*p} : Y_p \to Y_{p^*}, \, \omega_{p^*\chi(p^*)} : X_{\chi(p^*)} \to Y_{p^*}$ , then there exists a morphism  $h_{\chi(p^*)\chi(p)} : X_{\chi(p)} \to X_{\chi(p^*)}$ , and the following diagram is commutative:

(3) if  $h_{\chi(p^*)\chi(p)} : X_{\chi(p)} \to X_{\chi(p^*)}$ ,  $\omega_{p\chi(p)} : X_{\chi(p)} \to Y_p$ ,  $\omega_{p^*\chi(p^*)} : X_{\chi(p^*)} \to Y_{p^*}$ , then there exists a morphism  $h_{p^*p} : Y_p \to Y_{p^*}$  and the diagram  $(\Phi)$  is commutative.

It follows from condition (3) and the definition of a Hausdorff spectrum that, for example, every diagram



<sup>©</sup> Smirnov E. I., 2010

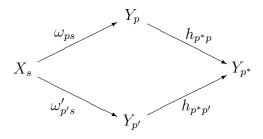
is commutative.

In particular, if  $|\mathfrak{F}| = |\mathfrak{F}^1| = \mathbb{Z}$  ( $\mathbb{Z}$  is the set of whole numbers),  $\mathfrak{F} = \{|\mathfrak{F}|\}, \mathfrak{F}^1 = \{|\mathfrak{F}^1|\},$ then we obtain a mapping of inverse spectra. Moreover, V. P. Palamodov's version of mapping of spectra [1] is a mapping of Hausdorff spectra. In fact, for each  $\beta \in \mathbb{Z}$  there exists the largest  $\alpha = \alpha(\beta) \in \mathbb{Z}$  such that  $(\alpha, \beta) \in \Delta_U$ , i.e. the inverse function of  $\alpha(\beta)$  from Condition II of Definition 2 in [1] defines a set of morphisms  $u_{\alpha}^{\beta} : X_{\alpha(\beta)} \to Y_{\beta}$  of the category  $K \ (\beta \in \mathbb{Z})$  which satisfies Condition I – this corresponds to fulfilling (1) and (2).

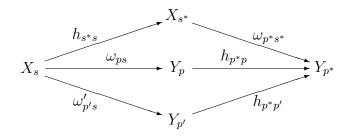
Suppose that  $\omega_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$  and  $\omega_{\mathcal{Z}\mathcal{Y}} : \mathcal{Y} \to \mathcal{Z}$  are mappings of Hausdorff spectra so that  $\omega_{\mathcal{Y}\mathcal{X}} = \omega(\varphi, \phi, \chi), \ \omega_{\mathcal{Z}\mathcal{Y}} = \omega(\varphi', \phi', \chi')$ . Let us put  $\varphi^* = \varphi' \circ \varphi, \ \phi^* = \phi \circ \phi', \ \chi^* = \chi \circ \chi'$ , so that  $\varphi^* : \mathfrak{F} \to \mathfrak{F}^2, \ \phi^* : F^2 \to F \ (\forall F \in \mathfrak{F}), \ \chi^* : T^2 \to T \ (\forall T^2 \in F^2)$ , setting  $\omega_{rs} = \omega_{rp} \circ \omega_{ps}$  whenever morphisms  $\omega_{ps}$  and  $\omega_{rp}$  are defined. It is easy to verify that the set of morphisms  $\omega_{rs} : X_s \to Z_r$  of the category  $\mathcal{G}$  satisfies conditions (1) and (2) for a mapping of Hausdorff spectra. We will call the mapping  $i : \mathcal{X} \to \mathcal{X}$ , where  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ , the *identity mapping* if it is formed by means of all the identity morphisms  $\omega_{ss} : X_s \to X_s \ (s \in |\mathfrak{F}|)$  of the category  $\mathcal{G}$ ; it is clear that i is a left and right identity under composition.

Thus, the set of Hausdorff spectra over  $\mathcal{G}$  and their mappings form a category, which (by analogy with [1]) we will denote by Spect  $\mathcal{G}$ . We may consider the category  $\mathcal{G}$  as a subcategory in Spect  $\mathcal{G}$  – namely, to each object  $A \in \mathcal{G}$  we assign the Hausdorff spectrum  $\mathcal{A} = \{A, \{A\}, \emptyset\}$ . Let  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\} \in \text{Spect } \mathcal{G}$ . Then every mapping  $\omega_{B\mathcal{X}} : \mathcal{X} \to B$  is given by a set of morphisms  $\omega_{Bs} : X_s \to B$ , where  $\varphi : \mathfrak{F} \to \{B\}, \phi_F : \{B\} \to F \ (\forall F \in \mathfrak{F}), \chi_F : \{B\} \to \phi_F(\{B\}) \text{ and}$  $s = \chi_F(\phi_F(\{B\})) \ (F \in \mathfrak{F})$ . Correspondingly, every mapping  $\omega_{\mathcal{X}A} : A \to \mathcal{X}$  is given by a set of morphisms  $\omega_{sA} : A \to X_s$ , where  $\varphi' : \{A\} \to \mathfrak{F}, \phi : F \to \{A\} \ (F = \varphi'(\{A\})), \chi : T \to \{A\}$  $(\forall T \in F), s \in |F|, F = \varphi'(\{A\}).$ 

Let  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\}, \mathcal{Y} = \{Y_p, \mathfrak{F}^1, h_{p'p}\}$  be objects from Spect  $\mathcal{G}$ . We will say that two mappings of Hausdorff spectra  $\omega_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$  and  $\omega'_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$  are *equivalent* if for any  $F \in \mathfrak{F}$ there exists  $F^* \in \mathfrak{F}^1$  such that the diagram



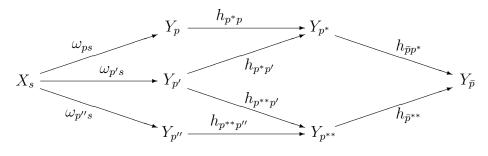
is commutative for any  $p^* \in |F^*|$   $(s \in |F|, p \in |\varphi(F)|, p' \in |\varphi'(F)|)$ . The relation introduced is reflexive, i.e. in this case  $p, p', p^* \in |F|, s = \chi(p), s = \chi(p'), s^* = \chi(p^*)$  and the following diagram is commutative because of  $(\Phi)$ :



The Category of Hausdorff Spectra over a Semiabelian Category

Specifically, the existence of a morphism  $h_{s^*s}$  of the Hausdorff spectrum  $\mathcal{X}$  follows from  $(\Phi)$ .

We now establish the transitivity of the relation. Let  $\omega_{\mathcal{YX}} \sim \omega'_{\mathcal{YX}}$  and  $\omega'_{\mathcal{YX}} \sim \omega''_{\mathcal{YX}}$ . Then transitivity follows from the directedness of the class  $\mathfrak{F}^1$  and the commutativity of the following diagram  $(\forall \overline{p} \in |\overline{F}|)$ :



Here,  $p \in |\varphi(F)|$ ,  $p' \in |\varphi'(F)|$ ,  $p'' \in |\varphi''(F)|$ ,  $p^* \in |F^*|$ ,  $p^{**} \in |F^{**}|$  and  $F^* \prec \overline{F}$ ,  $F^{**} \prec \overline{F}$ . It is clear that the equivalence relation is preserved under composition.

Thus the set  $\operatorname{Hom}(\mathcal{X}, \mathcal{Y})$  is decomposed into equivalence classes; let us now consider a new category  $\mathcal{H}$  whose objects are the objects of the category  $\operatorname{Spect} \mathcal{G}$ , while the set  $\operatorname{Hom}_{\mathcal{H}}(\mathcal{X}, \mathcal{Y})$  is formed by the equivalence classes of mappings  $\omega_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$ . We will denote these classes by  $||\omega_{\mathcal{V}\mathcal{X}}||$ .

Let  $\mathcal{G}$  be a semiabelian complete subcategory of the category TG, in which it is possible to construct direct sums and direct products. Then for each Hausdorff spectrum  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\}$  over  $\mathcal{G}$  there exists (as already shown) a unique (up to isomorphism) object of the category  $\mathcal{G}$ , called the *H*-limit of the Hausdorff spectrum  $\mathcal{X}$  and denoted by  $\lim_{\mathfrak{F}} h_{s's}X_s$ . Moreover, if

 $\omega_{\mathcal{YX}}: \mathcal{X} \to \mathcal{Y}$ , then there exists a unique morphism

$$\overline{\omega}_{\mathcal{YX}}: \underset{\overrightarrow{\mathfrak{F}}}{\overset{\longleftarrow}{\underset{\mathfrak{F}}}} h_{s's}X_s \to \underset{\overrightarrow{\mathfrak{F}}^1}{\overset{\longleftarrow}{\underset{\mathfrak{F}}}} h_{p'p}Y_p$$

of the category  $\mathcal{G}$ . In fact, let  $x \in \varprojlim_{\mathfrak{F} \in \mathfrak{F}} h_{s's}X_s$ , i.e.  $x \in \bigcup_{F \in \mathfrak{F}} \psi V_F^T$ , where  $\psi : \widehat{S} \to S$  is the canonical mapping and

$$V_F^T = \{ \alpha \in \prod_F X_s : x_s = \hat{h}_{s\hat{s}} x_{\hat{s}} , \ s, \hat{s} \in T \}.$$

Then there exists  $F \in \mathfrak{F}$  such that  $x \in \psi V_F^T$   $(T \in F)$ , and, consequently,  $x = \psi \alpha_T$ , where  $\alpha_T = (x_s^T)_F$ ,  $\alpha_T \in V_F^T$ ,  $T \in F$ . Therefore by the definition of a mapping of Hausdorff spectra there exist  $F^1 \in \mathfrak{F}^1$ ,  $F^1 = \varphi(F)$ ,  $\phi : F^1 \to F$  and  $\chi : T^1 \to T$   $(\forall T^1 \in F^1)$  which allow us to define a morphism of the category

$$g_{F^1F}:\prod_F X_s \to \prod_{F^1} Y_p\,,$$

where  $g_{F^1F} = \{\omega_{p\chi(p)}\}_{p\in|F^1|}$ . For each  $T^1 \in F^1$  we define an element  $\beta_{T^1} \in V_{F^1}^{T^1} \subset \prod_{F^1} Y_p$  such that  $\beta_{T^1} = \{\omega_{p\chi(p)} x_{\chi(p)}^T\}_{p\in|F^1|}$ , where  $T = \phi(T^1)$ . Here, given  $\hat{h}_{p\hat{p}} : Y_{\hat{p}} \to Y_p$ , there exists by  $(\Phi)$ 

 $\hat{h}_{\chi(p)\chi(\hat{p})}: X_{\chi(\hat{p})} \to X_{\chi(p)}$ , and moreover  $\hat{h}_{p\hat{p}}(\omega_{\hat{p}\chi(\hat{p})}x_{\chi(\hat{p})}^T) = \omega_{p\chi(p)}x_{\chi(p)}^T$ , where  $p, \hat{p} \in T^1$ . Now if  $\psi'$  is the canonical mapping for the Hausdorff spectrum  $\mathcal{Y}$ , then by  $(\Phi)$  we obtain  $\psi'\beta_{T_1^1} = \psi'\beta_{T_2^1}$  for arbitrary  $T_1^1, T_2^1 \in F^1$ . It remains to put  $y = \psi'\beta_{T^1}$   $(T^1 \in F^1)$ , where  $y \in \bigcap_{T^1 \in F^1} \psi' V_{F^1}^{T^1}$ , and, consequently,  $y \in \varinjlim_{\hat{y}^1} h_{p'p}Y_p$  and  $\overline{\omega}_{\mathcal{YX}}x = y$ . Additivity and continuity of  $\overline{\omega}_{\mathcal{YX}}$  are obvious and come directly from the definition of the *H*-limit of a Hausdorff spectrum, therefore  $\overline{\omega}_{\mathcal{YX}}$  is

and come directly from the definition of the *H*-limit of a Hausdorff spectrum, therefore  $\overline{\omega}_{\mathcal{YX}}$  is a morphism of the category *TG*. We will employ the notation  $H(\omega_{\mathcal{YX}}) = \overline{\omega}_{\mathcal{YX}}$ .

It is clear that H translates the identity mapping into the identity and a composition of mappings into a composition. Therefore H is a covariant functor from the category Spect  $\mathcal{G}$  into the category  $\mathcal{G}$ . Moreover, we have the following result:

**Proposition 1.** Let H: Spect  $\mathcal{G} \to \mathcal{G}$ . Then H can be extended to the category  $\mathcal{H}$  and is additive on it.

**Proof.** We show first of all that  $\operatorname{Hom}_{\mathcal{H}}(\mathcal{X}, \mathcal{Y})$  is an abelian group. Let  $\omega_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$ ,  $\omega'_{\mathcal{Y}\mathcal{X}} : \mathcal{X} \to \mathcal{Y}$ , where  $\mathcal{X}, \mathcal{Y} \in \operatorname{Spect} \mathcal{G}$ ,  $\omega_{\mathcal{Y}\mathcal{X}} = \omega(\varphi, \phi, \chi)$ ,  $\omega'_{\mathcal{Y}\mathcal{X}} = \omega'(\varphi', \phi', \chi')$ . For each  $F \in \mathfrak{F}$ we can find  $F^* \in \mathfrak{F}^1$  such that  $\varphi(F) \prec F^*$  and  $\varphi'(F) \prec F^*$ . Let us construct mappings of Hausdorff spectra  $\overline{\omega}_{\mathcal{Y}\mathcal{X}}^{1,2} : \mathcal{X} \to \mathcal{Y}$  so that  $\omega_{\mathcal{Y}\mathcal{X}} \sim \overline{\omega}_{\mathcal{Y}\mathcal{X}}^1$  and  $\omega'_{\mathcal{Y}\mathcal{X}} \sim \overline{\omega}_{\mathcal{Y}\mathcal{X}}^2$ ,  $\bigcup_{F^*} \overline{\chi}^1(p) = \bigcup_{F^*} \overline{\chi}^2(p)$ . In fact, for  $p \in |F^1|$  there exists  $s_p \in |F|$  such that  $\hat{h}_{\chi(p)s_p} : X_{s_p} \to X_{\chi(p)}$ ,  $\hat{h}_{\chi'(p)s_p} : X_{s_p} \to X_{\chi'(p)}$ and moreover, if  $p \in T^*$ ,  $\omega_{FF^*} : F^* \to F$ ,  $\omega_{F'F^*} : F^* \to F'$ , then  $s_p \in T$ , where  $T = \overline{\phi}(T^*)$ ,  $T \supset \phi[\omega_{FF^*}(T^*)]$ ,  $T \supset \phi'[\omega_{F'F^*}(T^*)]$ . Putting  $\overline{\chi}^1(p) = s_p$  and  $\overline{\chi}^2(p) = s_p$  in this case, we obtain the necessary identity. Now if we put  $\overline{\varphi}(F) = F^*$ , then the mappings of Hausdorff spectra  $\overline{\omega}_{\mathcal{Y}\mathcal{X}}^1 = \omega(\overline{\varphi}, \overline{\phi}, \overline{\chi}^1)$ ,  $\overline{\omega}_{\mathcal{Y}\mathcal{X}}^2 = \omega(\overline{\varphi}, \overline{\phi}, \overline{\chi}^2)$  are equivalent to  $\omega_{\mathcal{Y}\mathcal{X}}$  and  $\omega'_{\mathcal{Y}\mathcal{X}}$  respectively by  $(\Phi)$ . Therefore we define  $||\omega_{\mathcal{Y}\mathcal{X}}|| + ||\omega'_{\mathcal{Y}\mathcal{X}}||$  to be the element of  $\operatorname{Hom}_{\mathcal{H}}(\mathcal{X}, \mathcal{Y})$  containing

$$\{\omega_{p\overline{\chi}^1(p)} + \omega_{p\overline{\chi}^2(p)}\}_{p\in |F^*|} \quad (F\in\mathfrak{F}, \ F^* = \overline{\varphi}(F)).$$

Clearly, this class does not depend on the choice of representatives  $\omega_{\mathcal{YX}}$ ,  $\omega'_{\mathcal{YX}}$  in their equivalence classes. The operation of addition which has been introduced converts  $\operatorname{Hom}_{\mathcal{H}}(\mathcal{X}, \mathcal{Y})$  into an abelian group. Now the extension of the functor H to the category  $\mathcal{H}$  and its additivity there are obvious. The proposition is proved.

We will reserve the notation H = Haus for the case  $\mathcal{G} = TLC$ .

We introduce a semiabelian structure on the category  $\mathcal{H}$ . For any objects  $\mathcal{X}, \mathcal{Y}, \mathcal{Z} \in \mathcal{H}$  the law of composition defines a bilinear mapping

$$\operatorname{Hom}_{\mathcal{H}}(\mathcal{X},\mathcal{Y}) \times \operatorname{Hom}_{\mathcal{H}}(\mathcal{Y},\mathcal{Z}) \to \operatorname{Hom}_{\mathcal{H}}(\mathcal{X},\mathcal{Z}).$$

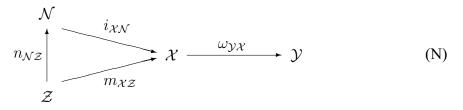
Thus  $\mathcal{H}$  is an additive category.

**Proposition 2.** (See [1].) The category  $\mathcal{H}$  is semiabelian.

**Proof.** Let  $||\omega_{\mathcal{YX}}|| : \mathcal{X} \to \mathcal{Y}$ , where  $\mathcal{X}, \mathcal{Y}$  are Hausdorff spectra over  $\mathcal{G}$ . We will construct for a morphism  $||\omega_{\mathcal{YX}}||$  of the category  $\mathcal{H}$  its kernel and cokernel. We choose in the class  $||\omega_{\mathcal{YX}}||$ some element  $\omega_{\mathcal{YX}} \in \text{Spect } \mathcal{G}$  so that  $\omega_{\mathcal{YX}} = \omega(\varphi, \phi, \chi)$ . Now for each  $s \in |F|$ , where  $F \in \mathfrak{F}$ , let us consider an object  $N_s \in \mathcal{G}, N_s \subset X_s$ , provided with the topology induced from  $X_s$ , and such that  $N_s = \ker \omega_{ps}$  for  $s = \chi(p)$   $(p \in |\varphi(F)|)$ . By  $(\Phi)$  the restriction  $n_{s's}$  of the morphism  $h_{s's}$ translates  $N_s$  into  $N_{s'}$ , therefore the family  $\mathcal{N} = \{N_s, \mathfrak{F}, n_{s's}\}$  is a Hausdorff subspectrum of the Hausdorff spectrum  $\mathcal{X} = \{X_s, \mathfrak{F}, h_{s's}\}$ . We will show that the identity embedding  $i_{\mathcal{XN}} : \mathcal{N} \to \mathcal{X}$  is the kernel of  $\omega_{\mathcal{YX}}$ . For this it is enough to establish that for any morphism  $m_{\mathcal{XZ}} : \mathcal{Z} \to \mathcal{X}$  of the category Spect  $\mathcal{G}$  such that  $\omega_{\mathcal{YX}} \circ m_{\mathcal{XZ}} = 0$ ,

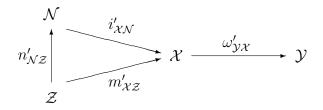
$$\mathcal{Z} \in \operatorname{Spect} \mathcal{G}, \quad \mathcal{Z} = \{Z_t, \mathfrak{F}^\circ, h_{t't}\}$$

there exists a morphism  $n_{NZ} : Z \to N$  of the category Spect G such that the following diagram is commutative:

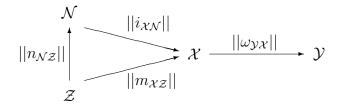


(Here the zero mapping of spectra signifies that for  $\omega_{\mathcal{YX}} = 0 = \omega_{\mathcal{YX}} \circ m_{\mathcal{XZ}}$  its component morphisms  $\omega_{pt} = \omega_{ps} \circ \omega_{st}$  are such that  $\omega_{pt}(Z_t) = 0$ .)

At the same time it is clear that, if for  $\omega'_{\mathcal{YX}} \sim \omega_{\mathcal{YX}}$ ,  $m'_{\mathcal{XZ}} \sim m_{\mathcal{XZ}}$  such that  $\omega'_{\mathcal{YX}} \circ m'_{\mathcal{XZ}} = 0'$ , where  $0' \sim 0$ , there also exist  $n'_{\mathcal{NZ}} \in \text{Spect } \mathcal{G}$  and  $i'_{\mathcal{XN}} \sim i_{\mathcal{XN}}$  such that the diagram



is commutative, then  $n'_{\mathcal{NZ}} \sim n_{\mathcal{NZ}}$ . Therefore, if diagram (N) applies, each morphism  $||\omega_{\mathcal{YX}}||$  of the category  $\mathcal{H}$  such that  $||\omega_{\mathcal{YX}}|| \circ ||m_{\mathcal{XZ}}|| = 0$ , where  $||m_{\mathcal{XZ}}|| : \mathcal{Z} \to \mathcal{X}$ , and  $i_{\mathcal{XN}} \in ||i_{\mathcal{XN}}||$ , has kernel  $||i_{\mathcal{XN}}||$  such that there exists  $||n_{\mathcal{NZ}}||$  with commutative diagram



Thus, for the existence of the kernel of the morphism  $||\omega_{\mathcal{YX}}||$  it is enough to establish the existence of  $n_{\mathcal{NZ}} : \mathcal{Z} \to \mathcal{N}$  and the commutativity of diagram (N).

If the mapping of spectra is  $m_{\mathcal{XZ}} = m(\varphi^{\circ}, \phi^{\circ}, \chi^{\circ})$ , then taking into account the fact that  $\operatorname{Im} \omega_{s\chi^{\circ}(s)} \subset N_s$   $(s \in |\phi^{\circ}(F^{\circ})|, F^{\circ} \in \mathfrak{F}^{\circ})$  by assumption, we can construct a mapping of Hausdorff spectra  $n_{\mathcal{NZ}} : \mathcal{Z} \to \mathcal{N}$ , where  $n_{\mathcal{NZ}} = n(\varphi^{\circ}, \phi^{\circ}, \chi^{\circ})$ , so that its constituent morphisms  $\widehat{\omega}_{s\chi^{\circ}(s)} : Z_{\chi^{\circ}(s)} \to N_s$  are restrictions of the morphisms  $\omega_{s\chi^{\circ}(s)}$ . Commutativity of the diagram is obvious.

Now we will construct the cokernel of the morphism  $||\omega_{\mathcal{YX}}||$ ; let  $\omega_{\mathcal{YX}} \in ||\omega_{\mathcal{YX}}||$ . For each  $p \in |\varphi(F)|$   $(F \in \mathfrak{F})$  let us consider the factor group  $R_p = Y_p/\mathrm{Im}\,\omega_{p\chi(p)}$  with the topology

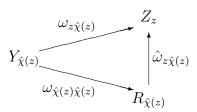
induced from  $Y_p$ . It is clear that because of  $(\Phi)$  the subgroups  $\operatorname{Im} \omega_{p\chi(p)}$  form a Hausdorff spectrum, therefore the factor groups  $R_p$   $(p \in |\varphi(F)|)$  also form a Hausdorff spectrum; let

$$\mathcal{R} = \{R_p, \mathfrak{F}_0^1, h_{p'p}\}, \quad \mathcal{Y}_0 = \{Y_p, \mathfrak{F}_0^1, h_{p'p}\},$$

where  $\mathfrak{F}_0^1 = \mathfrak{F}^1|_{\varphi(\mathfrak{F})}$  (without loss of generality we may assume that  $\varphi(\mathfrak{F}) = \mathfrak{F}^1$ ). Let us denote by  $\omega_{\mathcal{R}\mathcal{Y}} : \mathcal{Y} \to \mathcal{R}$  the canonical mapping of Hausdorff spectra; we will show that  $||\omega_{\mathcal{R}\mathcal{Y}}||$ is the cokernal of the morphism  $||\omega_{\mathcal{Y}\mathcal{X}}||$ . For this it follows that we have to establish that, for any morphism  $m_{\mathcal{Z}\mathcal{Y}} : \mathcal{Y} \to \mathcal{Z}$  of the category Spect  $\mathcal{G}$  such that  $m_{\mathcal{Z}\mathcal{Y}} \circ \omega_{\mathcal{Y}\mathcal{X}} = 0$ , there exists a morphism  $n_{\mathcal{Z}\mathcal{R}} : \mathcal{R} \to \mathcal{Z}$  of the category Spect  $\mathcal{G}$  such that the following diagram is commutative:

$$\mathcal{X} \xrightarrow{\omega_{\mathcal{Y}\mathcal{X}}} \mathcal{Y} \xrightarrow{m_{\mathcal{Z}\mathcal{Y}}} \stackrel{\mathcal{Z}}{\uparrow} n_{\mathcal{Z}\mathcal{R}} \qquad (K)$$

If  $m_{\mathcal{Z}\mathcal{Y}} = m(\widehat{\varphi}, \widehat{\phi}, \widehat{\chi})$ , then  $n_{\mathcal{Z}\mathcal{R}} = n(\widehat{\varphi}, \widehat{\phi}, \widehat{\chi})$ , and since  $\omega_{z\widehat{\chi}(z)}(Y_{\widehat{\chi}(z)}) = 0$  for all  $z \in |\widehat{\varphi}(F^1)|$  $(F^1 \in \mathfrak{F})$ , then  $\operatorname{Im} \omega_{\widehat{\chi}(z)\chi(\widehat{\chi}(z))} \subset N_{\widehat{\chi}(z)}$ , and, consequently, because the category  $\mathcal{G}$  is semiabelian there exists a morphism  $\widehat{\omega}_{z\widehat{\chi}(z)} : R_{\widehat{\chi}(z)} \to Z_t$  such that the following diagram is commutative:



Thus, as is not difficult to see, the set of morphisms  $\hat{\omega}_{z\hat{\chi}(z)}$  defines a mapping of Hausdorff spectra in such a way that diagram (K) is commutative. The proposition is proved.

## Библиографический список

- 1. Palamodov V.P.: Functor of projective limit in the category of topological linear spaces. Math. Collect., V.75. N4 (1968), P.567–603.
- 2. Palamodov V.P.: Homological methods in the theory of locally convex spaces. UMN., V.26. N1(1971), P.3–65.
- Smirnov E.I. Hausdorff spectra in functional analysis. Springer-Verlag, London, 2002. 209p.
- 4. Zabreiko P.P., Smirnov E.I.: On the closed graph theorem. Siberian Math. J., V.18. N.2 (1977), P.305–316.
- 5. Rajkov D.A.: On the closed graph theorem for topological linear spaces . Siberian Math. J., V.7. N2 (1966), P.353–372.

- 6. Smirnov E.I.: On continuity of semiadditive functional. Math. Notes., 1976. V.19. N4 (1976), P.541–548.
- Wilde M. : Reseaus dans les espaces lineaires a seminormes. Mem. Soc. Roi. Sci. Liege., V.19. N4 (1969), P.1–104.
- 8. Smirnov E.I.: The theory of Hausdorff spectra in the category of locally convex spaces. Functiones et Aproximatio, XXIV. UAM, 1996, P.17–33.
- 9. Retakh V.S.: On dual homomorphism of locally convex spaces. Funct. Anal. and its Appl., V.3. N4 (1969), P.63–71.
- 10. Nobeling C. : Uber die Derivierten des Inversen und des Directen Limes einer Modulfamilie. Topology I., 1962, P.47–61.
- 11. Cartan A., Heilenberg C. Homological algebra. M.: Mir. 1960, 510p.
- 12. Smirnov E.I. Hausdorff spectra and the closed graph theorem. In: Pitman Research Notes in Mathematics Series, Longman, England, 1994. P.37–50.