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Founding of Mathematical Structures Through Learners' Network Project Activities

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A teacher of mathematics does not only provide learners with new knowledge, ideas, methods and technologies but also creates conditions for the formation and development of key competences and universal educational actions targeted at more efficient ways of solving educational tasks. The Internet serving as an accessible source of information including educational content as well as appearing for the medium of social communication and self-realization, becomes a social and cultural environment in which individual users and user communities present themselves and function united by common goals and interests of networking and online interaction. Openness and ability to provide access to general information resources to all learners; implementation of students' joint productive activity and conducting dialogue of cultures through distribution and permanent exchange of Mathematics, Science, Arts and Humanities online resources; shaping student's individual opinions; ensuring a qualitatively new level of interaction of educational process subjects (horizontal interaction); gaining experience of reflection and collective action; transition from training to self-education – all this allows us to declare a need for and possibility of creating network educational communities which can be attributed to complex non-linear information systems featuring the tendency to self-organization and obeying the laws of synergetics in the processes of mastering and visual modeling of mathematical structures. Actualization and founding (Fundierung) of mathematical structures in their integrative and meaningful contexts seem a structure-forming factor of efficient mastering of the mathematical procedures in network communities.

Keywords: founding (Fundierung) of knowledge, visual modeling, network communities, mathematical structures, information-communication environment, teacher of mathematics.

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Фундирование математических структур в процессе обучения в сетевых сообществах студентов вузов

В статье представлены технологии обучения математическим структурам, когда учитель математики не только осваивает вместе с учениками новые знание, идеи, методы, но и создает условия для формирования и развития ключевых знаний и универсальных учебных действий обучающихся, предназначенных для применения более эффективных способов решения образовательных задач. Интернет, служащий доступным источником информации, включая образовательное содержание, а также являющийся средством социальной коммуникации и самореализации, становится интерактивной средой, в которой отдельные пользователи и пользовательские сообщества имеют возможность самовыражения, причем их объединяют общие цели и интересы освоения предметной деятельности в сетевом взаимодействии онлайн. Такими технологиями обеспечиваются открытость коммуникации и способность обеспечить доступ к ресурсам общей информации всем ученикам; внедрение производительного диалога культур в совместной деятельности в ходе освоения информации и обмена ею в области математики, естественных и гуманитарных наук; формирование и актуализация индивидуальных предпочтений и мнений студентов; обеспечение качественно нового уровня взаимодействия образовательных предметов и процессов (горизонтальное взаимодействие); приобретение опыта отражения и коллективного взаимодействия; переход от приемов и методов обучения до самообразования – все это позволяет актуализировать потребность в сетевых образовательных сообществах и возможность их создания и эффективного функционирования. Последние могут быть классифицированы как сложные нелинейные информационные системы, показывающие тенденцию к самоорганизации и подчиняющиеся законам синергетики в процессах освоения и наглядного моделирования математических структур. Таким образом, актуализация и процессы фундирования математических структур в их интегральных и значащих контекстах сетевого взаимодействия могут привести к эффективному формированию и развитию профессиональных компетенций и универсальных учебных действий обучающихся.

Ключевые слова: фундирование опыта личности, наглядное моделирование, сетевые сообщества, математические структуры, диалог культур, подготовка учителя математики.

Methodology and teaching experience

Web 2.0 services providing users with an opportunity of both retrieving information and creating their own information resources appear to be a powerful tool for learning the world and a most significant future-oriented means of education. In this regard, the mathematical education of prospective teachers of mathematics proves efficient if it involves an exchange of information and knowledge in the information-communication environment which provides active interaction of the education space participants, an open access to wide-ranging and professionally

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significant data, documentation of the educational process results (Kuznetsova, A. S. Tikhomirov, Kytmanov, S. A. Tikhomirov, Troshina, 2014). It is important that particular complexes of generalized information enabling to operate various practice-oriented knowledge in an integrative connection with the operation of information environments be selected from the content of Mathematics.

Mathematical structures are an important integrative construct of Mathematics. Their employment helps systematize Mathematics, at least to a certain extent (which is especially important for a prospective teacher of mathematics). Thus, V. A. Testov claims that «the idea of structures reflected (and having turned out very productive) in N. Bourbaki's multivolume treatise as well as the accordance between mathematical structures and the structures of human thinking discovered by the school of J. Piaget served as motivating factors for a radical reform of mathematical education in the 1960–70-ies in schools and universities both in this country and abroad... A significant drawback that showed up in the master strategy of Mathematics teaching in the course of the reform was the fact that basing themselves on some selected results of J. Piaget's research the scientist modernizers limited the course of Mathematics to the algebraic, order and topological structures without paying due attention to other kinds of mathematical structures (combinatorial, algorithmic, image-geometric etc.) which play a special role in research activities, in the formation of new conceptual structures» (Testov, 1999). Mathematical structures are defined and studied on the foundation of classical Mathematics – the language of the theory of sets. According to J. Piaget, the main types of mathematical structures are fundamental not only for building up Mathematics, but also for stimulating the mechanism of thinking (Piaget, 1960). According to N. Bourbaki (Bourbaki, 1963), the mathematical structure is a system which can be described through $S = < M; R1, R2, \ldots, Rk >$, where M is the basic set, R1, R2,…, Rk are defined relations whose properties are determined by axioms. They described three main types of mathematical structures: algebraic, topological and order structures. These types form the structural framework but do not exhaust all Mathematics. In addition to these types of mathematical structures there also exist projective, metric, combinatorial, image, logical, algorithmic, probabilistic, and stochastic schemes (M. A. Kholodnaya, V. A. Krutetskiy, I. J. Kaplunovich, L. B. Itelson, etc.).

Groups are the most important type of algebraic structures. They reflect the fundamental property of symmetry. Another important type includes structures that are defined by the relation of order, formalizing the idea of magnitude comparison of objects (elements of the set). Order structures are an integral part of discrete Mathematics and form the mathematical core of computer science. The order relation is studied in the theory of ordered sets and lattices. Unlike algebraic and topological structures, order structures are not studied at school or university in their explicit form. Consequently, it makes sense to include order structures in the content of optional courses or do some preliminary study of their basic concepts, e.g., ordered sets, within various mathematical courses. Thus, in the process of studying algebra and theory of numbers the attention of the prospective mathematics teacher should be called to the examples of the general order operations of sup and inf (union and intersection of sets); the LCM (least common multiple) and the GCD (greatest common divisor) of natural numbers. For the course of mathematical logic such examples are the conjunction and disjunction of statements. Different ordered structures present a rich material for the development of educational network projects.

The third important type of structures, topological, is expressed in the concept of topological space that formalizes the abstractions of continuity and limiting transition. Topological structures serve the framework of continuous Mathematics: mathematical analysis, geometry, topology. Mathematical ideas of measurability and integrability are reflected in the theory of measures and integration whose foundation is based on the concepts of measure space and metric space.

The above mathematical cognitive structures are referred to the systems of knowledge storage, while logical, combinatorial, algorithmic, image-geometric, probabilistic and stochastic schemes belong to the means of cognition. Logical schemes allow to draw correct conclusions from true assumptions, get a proper corollary from the given facts, construct the whole from the given parts with predetermined properties. Algorithmic schemes enable both to apply the known algorithms and methods and formulate and build new ones. Image-geometric schemes allow to visually interpret abstract mathematical objects as well as operate visual images and concepts. Probabilistic schemes are aimed at identifying differences and establishing casual relationship and consistency between different mathematical objects which are reflections of real phenomena and processes and provide their assessment and forecasting. Using probabilistic schemes the prospective teacher of mathematics can distinguish a variable situation from all others and predict the course of its further development. To the mathematical structures of discrete Mathematics there belong the algebra of formal power series in enumerative combinatorics, propositional algebra, the algebraic theory of algorithms (Zhuravlev, 1974).

The innovative concept of founding (Fundierung) developed by E. I. Smirnov (Smirnov, 2012) is an effective mechanism for enhancing the theoretical and practical components of mathematical education of the prospective mathematics teacher taking the structural role in the functional model. The basis of this concept is alteration of the content and structure of mathematical and methodical training in the direction of enhancing the professional component of mathematical education with the subsequent founding (Fundierung) of knowledge and experience at different levels. Targeted steps taken to build the founding (Fundierung) spirals of the basic educational (mathematical) elements seem essential within the framework of this concept. We find it reasonable to use mathematical structures as the mathematical material for the spiral modeling of the basic knowledge and skills in Mathematics and mathematical education techniques of the prospective teacher of mathematics.

Mathematical structures can be a structure-forming factor allowing to select the basic theoretical and practical information from the content of different mathematical courses through which the founding (Fundierung) of school knowledge and teaching experience occurs. The concept of founding (Fundierung) implies that the essence of mathematical structures reveals itself in its various practice-oriented manifestations gradually rising to a higher level of abstraction. The founding (Fundierung) of individual experience becomes especially important in the modern period on the back of an increasing tendency to the development of one's motivational sphere, metacognitive experiences, processes of self-actualization and self-realization alongside the deployment of adequate pedagogical conditions, the subject content, tools, forms and educational technologies essential for teaching Sciences and Humanities. The founding (Fundierung) procedures of transition from the initial state of an entity and its actual view to the generalized potential development of the entity into a perfect object (process or phenomenon, state of personal qualities) are multi-stage, multifunctional, targeted and integrative for the actualization of the intra- and interdisciplinary connections. In such event the procedures of transition are more distinct and targeted in the zones of proximal development if the indicative and information bases of the learners' educational activity are cemented with the specially designed content of the course which is visually modeled in the form of founding (Fundierung) spirals or clusters of the basic educational elements (Shadrikov, Smirnov, 2002). Thus, the founding (Fundierung) chain of entities forms a founding (Fundierung) spiral as an integrative construct of visual modeling of the phased implementation of continuity of school and university education content and formation of personality traits from school characteristics to the professional competencies of the prospective teacher. However not only one's individual experience but one's individual mental functions and intelligent operations can be formed holistically and purposefully on the basis of the founding (Fundierung) concept. The development of the latter is very important (e.g., in the context of development of abilities) and following V. D. Shadrikov (Shadrikov, 2009), we will define an intellectual operation as «a set of conscious mental activities associated with cognition and solution of tasks faced by the individual». Intellectual operations (both cognitive and metacognitive) determine the content of universal educational actions such as modeling, goal-setting, planning, analysis, synthesis, analogy, etc.

Methods and procedures

We see it reasonable to instruct the prospective mathematics teacher on the mathematical structures in an information-communication environment (ICE). The ICE functional model (Fig. 1) is the information environment closest to the subject, a set of tools and conditions in which his [her] educational activity and personality formation occur. In the center of the ICE the prospective teacher of mathematics (the active subject of the environment) is to be found: his motivational attitudes, psychological characteristics and cognitive needs. Person-centered education that combines such educational technologies as founding (Fundierung), visual modeling, contextual learning, cooperative learning, project learning is the focus area of the ICE.

The latter of these technologies is viewed as a priority because it counts on the independent activity of the prospective teacher of mathematics, which results in the development of his [her] cognitive skills, ability to structure and actualize knowledge, to define and solve problems.

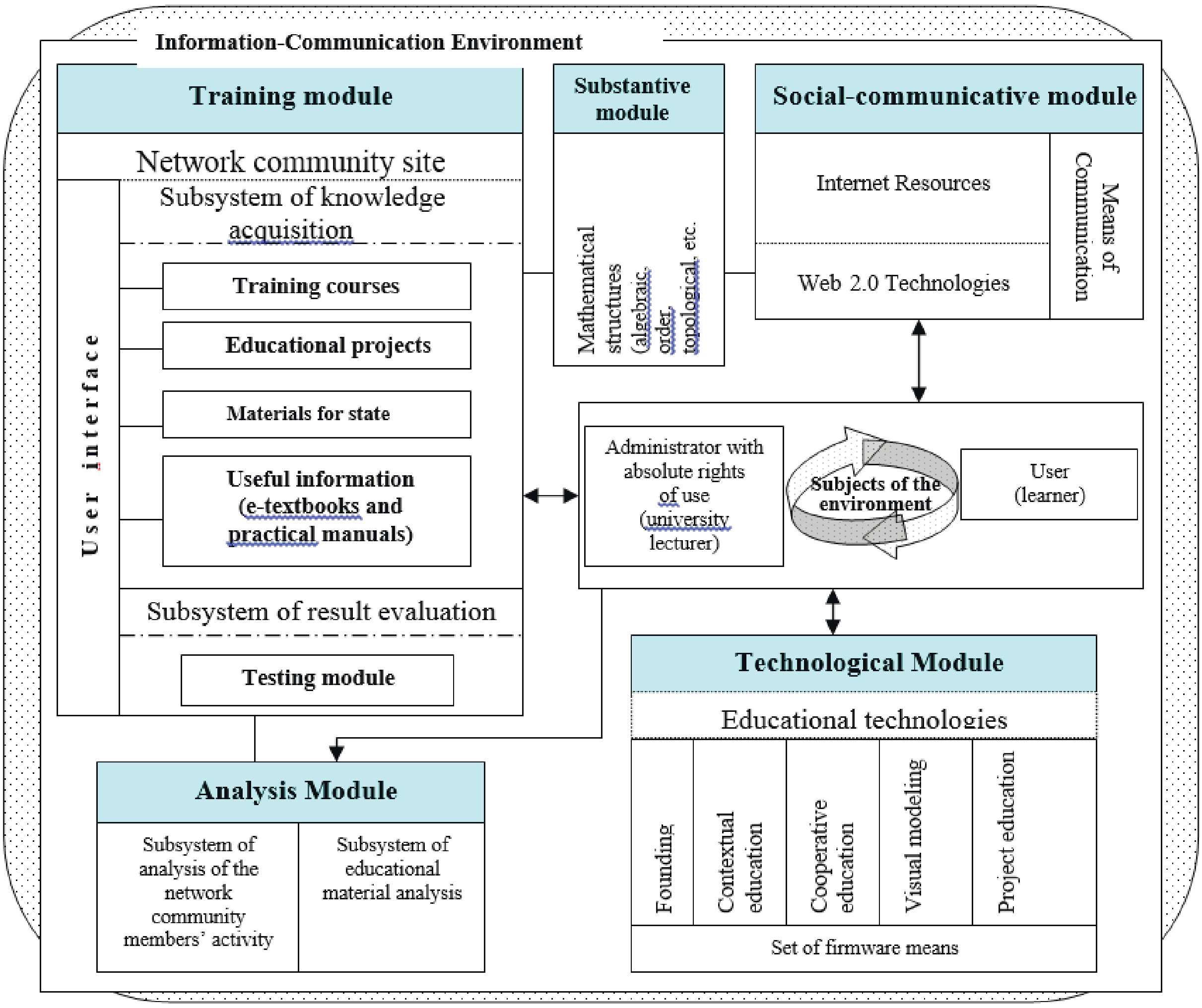

Figure 1: ICE Functional model

The founding (Fundierung) of individual experience is seen as a necessary construct for the development of learners' intellectual operations from the actualization of the initial state of experience and its particular manifestations on the basis of a detailed variation and analysis (synthesis) of the situation to further theoretical understanding on the basis of divergent thinking and visual modeling, to the implementation of solutions of specific tasks in the context of the extension and saturation of the information environment with the use of the ICT-support.

The founding (Fundierung) spiral depicted in Fig. 2 forms the indicative basis of the educational activity in the process of studying the concept of the mathematical structure. Teachers' and learners' activities which deploy and develop throughout the educational process on the basis of the education content are central in any methodical system. Studying the mathematical structures the prospective teacher of mathematics must acquire knowledge, understanding, skills of application of the basic entity structure constructs as well as the innovative teaching experience targeted at the formation and development of the relevant competence. This is possible if, along with the traditional educational methods, the prospective teacher uses the project method in the Web 2.0 services, particularly in the Wiki environment, which will facilitate the formation of the prospective teacher's universal educational actions.

The educational network project is a form, means and set of actions cooperatively performed in a specific sequence by the learners' and teachers' communities on the Internet. It is aimed at working out an independent solution of a particular problem (task) that is meaningful to the learners and results in a finalized innovative product. Implementation of an educational project on the Internet actually means creation of a network community. Therefore we can consider the network project activity of prospective mathematics teachers as an activity of the network community. The dominant activity taken as the project basis, we can distinguish the following types of educational projects (e. g., focused on mathematical structures) implemented in the Wiki environment: information retrieval, educational research, creative, practice-oriented and methodical projects. The information retrieval project may set a topic «Creating a database of the basic algebraic structures». The database must include the name of the algebraic structure, its designation, definition, criteria, examples, properties and basic theorems, data on how the algebraic structure is used in various areas of Mathematics, historiogenesis of the structure content and applications development, construction of graphs of matching algebraic structures to mathematical knowledge etc. Through an independent research that focuses on the analysis of the basic algebraic structures learners pass the evaluation stage and are expected to answer, e. g., the following questions: «Do the examples of rings listed in the database refer to fields?», «Does the ring (field) contain an element neutral in relation to the multiplication operation?», «What theorems, correct for the field, are also correct for the ring?», «How are groups, rings and fields interconnected?», «What properties of the rings are not relevant for the groups?». The question-answer session is limited in time.

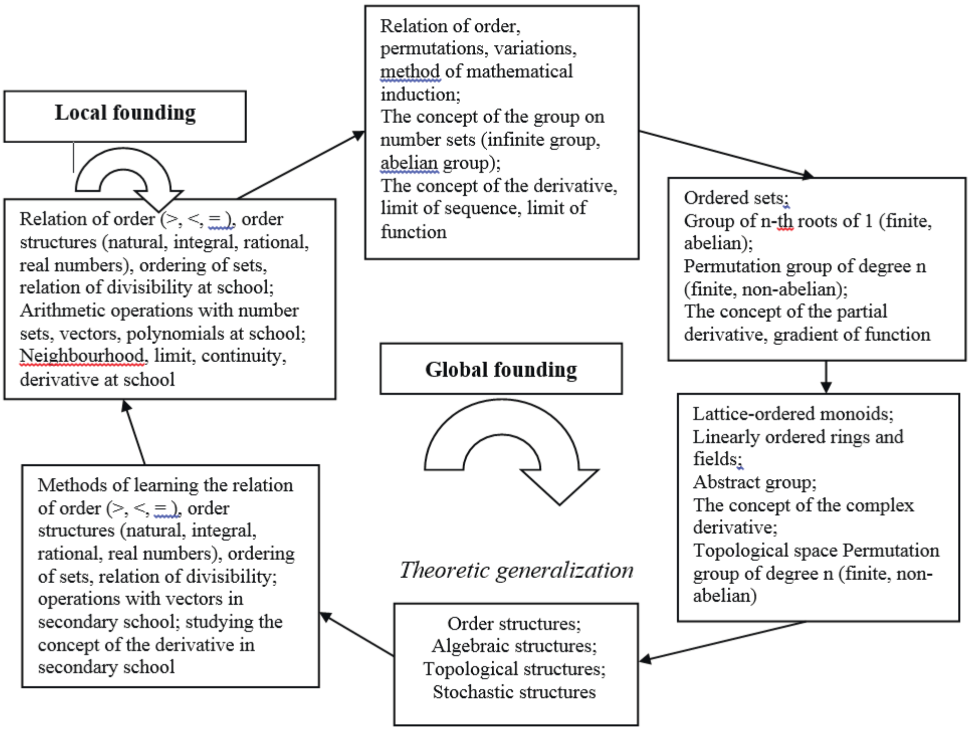

Figure 2: Founding (Fundierung) spiral of the content of mathematical structure

The practice-oriented project is a type of educational project aimed at the solution of practice-oriented tasks and featuring a specific content of the learners' professional actions. The result of such a project implies acquisition of particular competences of a teacher. Projects involving investigations of the algebraic methods penetration into various fields of Economics or Natural Science can serve good examples of a practice-oriented project: e. g., «Application of group theory to the study of regularities of symmetry». The implementation of this project requires searching for solutions to the chains of interdisciplinary practice-oriented tasks.

Below we present some examples of chains of practice-oriented tasks.

Chain 1

1. Find all n-element lattices for *n*6.

2. Find a group *G* of automorphisms (symmetries) of the 5-element lattice with three atoms and draw up a Cayley table for this group.

3. Find the orders of all group *G* elements.

The groups are called isomorphic (equal) if their elements can be labeled in such a way that they have the same Cayley table.

4. Find all subgroups of *G* and specify those among them that are isomorphic to two-element groups of symmetries of a regular triangle.

5. Find left and right cosets of all subgroups of the group *G*.

6. Prove that *G* is not abelian.

7. Find all six-element lattices, which have non-trivial symmetries (non-identical automorphisms).

In physics, the group of symmetries of an object appears as the group of its coordinate transformations that leave the structural properties of the object unchanged.

8. Prove that *G* is isomorphic to the group of symmetries of the sphere.

9. Find a 7-element lattice whose automorphism group is isomorphic to the symmetric group of five-element permutations.

10. Find a lattice whose symmetry group is isomorphic to the symmetry group of the tetrahedron or the symmetry group of the CH_4 molecule spatial configuration (the task implements interdisciplinary links between abstract algebra, geometry and chemistry).

Chain 2

1. Find all group elements of the symmetric group S_4.

2. Find all elements of the symmetry group of the tetrahedron.

3. Prove that the symmetry group of the tetrahedron is isomorphic to the symmetric group S_4.

4. Find a subgroup of the symmetric group S_4 that is isomorphic to the group of rotations of the tetrahedron.

5. Find a subgroup of the symmetry group of the tetrahedron that is isomorphic to the group of third roots of unity under multiplication.

6. Does the symmetry group of the tetrahedron contain a subgroup isomorphic to the module-3 residues addition group?

7. Find a subgroup of the symmetry group of the tetrahedron that is isomorphic to the automorphism group of the directed graph in Fig. 3.

8. Find a subgroup of the symmetric group S_4 that is isomorphic to the automorphism group of the lattice in Fig. 4 (called the diamond).

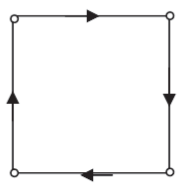

Figure 3: Directed graph

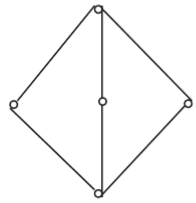

Figure 4: Lattice

9. Prove that the alternating subgroup of degree 4 of the symmetric group S_4 is a normal divisor of the group.

It is known that the cube or the octahedron is one of the main forms of the diamond crystal.

10. Find all the elements of the symmetry group of the cube.

11. Prove that the group of symmetries of the cube and the group of symmetries of the octahedron are isomorphic to each other.

12. Is the symmetric group S_4 a subgroup of the symmetry group of the cube?

13. Find an undirected graph whose automorphism group is isomorphic to the symmetry group of the tetrahedron.

14. There are a hundred stones. Two players in turns take from 1 to 5 stones. The player who takes the last stone loses. Find the element of the module-6 residues addition group which provides a winning strategy for the first player.

15. Find a 5-element lattice whose automorphism group is isomorphic to the group of symmetries of the curve called the folium of Descartes.

To carry out the «Application of group theory to the study of regularities of symmetry» educational network project students are divided into teams, each solving its chain of tasks presented on the website of the network community.

Here we would like to outline *the work stages* of the educational network projects devoted to the theory of mathematical structures:

– *organizational-preparatory stage*: choosing the project topic, goal setting and assignment of tasks, discussion of possible sources of information; actualization of the current knowledge and anticipation of potential mathematical knowledge to be obtained in the course of planning and implementing the educational networking activity: practice-oriented levels of acquisition of the basic educational elements of mathematical structures; historiogenesis and personalities determined by the selected topic; application-oriented historical tasks leading to the educational elements; motives, conditions and driving forces of practice efficiency preceding the emergence of mathematical structures; selection and variety of forms and methods of information representation: probabilistic and statistical, content, graphics, cluster; collection and mathematical analysis of data;

− *substantive-technological*: students' independent work in accordance with the tasks set, discussion of the project intermediate results with the project partners; research and creative activity in solving problems: data acquisition, knowledge transfer, extension and testing of hypotheses, reflection, visual modeling, process orientation; formalization of the various levels of modeling (conceptual, scientific, mathematical, informational, integrative); visual continuity, visual-graphic association; construction of visual models of various sign modalities, verification and self-control of the adequacy of the author's (learner's) solutions, assessment of knowledge; determination of regularities, analogies, associations, dynamics of the processes under investigation, phenomena and facts; actualization of the plurality of solutions based on the uniqueness of data; intuition and result forecast;

− *evaluative*: formation and development of *integrative constructs of intellectual operations* (modeling, understanding, planning, forecasting, decision making) as development mechanisms based on diagnostics and deployment of dialogue- and practice-oriented founding (Fundierung) procedures aimed at the solution of particular, specific tasks through resource interaction and increase of independence and responsibility for the decisions made (including volitional and moral aspects) in the transition from thinking to action; searching for and an algorithm of reaching a solution, making a decision, insight, documentation and verification of procedures and algorithms, presentation of results, defense of the educational project, panel discussion, progress review.

Results

Grades for the educational projects are given during the last stage using the point-rating system intended for self-evaluation, assessment of the project team leader and the university lecturer. A 3-grade scale is employed for assessment: 3 points are given to the project participant if the skill is well formed, 2 points – if the skill is medium level, 0 points – if the skill is not formed.

The maximum possible score that each participant can get is 99. The score correlates with the traditional assessment system within the following range:

− 85–9 points – «excellent»;
− 70–84 points – «good»;
− 50–69 points – «satisfactory»;
− below 50 points – «unsatisfactory».

The proposed system of project evaluation stimulates a responsible attitude of prospective teachers of mathematics to its implementation.

Here we list the teaching methods used for the organization of the educational activity of prospective mathematics teachers in an network community and aimed at the formation of their methodical competence:

− the content of tasks for educational network projects should be practice-oriented and selected in accordance with the principles of interdisciplinarity and dialogue on the basis of visual modeling of founding (Fundierung) constructs. The task solutions must be repeatedly reviewed, given thought to and complemented;

− some informative-didactic work must be carried out over the mathematical task at the organizational-preparatory stage of the educational network project: it is essential to ensure that the prospective teacher of mathematics analyzes the problem and defines the objectives; singles out the theoretical knowledge required for the problem solution (identifies the subject area); discloses the content of the theoretical data contained in the problem; chooses the means of solving the problem; analyzes and evaluates the results of solving the problem.

− the substantive-technological stage requires solution of the educational, methodical tasks targeted at the prospective mathematics teacher mastering the teaching techniques appropriate for working in the network community.

Conclusion

In the course of implementation of the educational network projects focused on the study of mathematical structures the prospective teachers (and this is extremely important for their future career) gain the experience of carrying out project activities in the Internet educational communities which is a special form of academic work aimed at fostering the teacher's self-sufficiency, responsibility and professional competence. The form of project activities varies from the educational academic type (in which the traditional procedures of obtaining and revising the educational material are reproduced, the basic unit of the learner's activity being a speech act) to the educational-professional type which combines features of the educational and future professional activity.

Fulfillment of the educational activity in the network educational community based on the focused, productive and professionally-oriented interaction of the subject and the didactical and communicative opportunities of the network community with the aim of mastering the mathematical structures as the founding (Fundierung) constructs of the school knowledge contributes to the formation and development of the methodical competence of the prospective mathematics teacher.

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